Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

0350543551

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Solve the equation $5^{w-1} = 12$, giving your answer correct to 2 decimal places. [2]

(b) Solve the equation $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$. [3]

2 **(a)** Write $2\lg x - \left(\lg(x+6) + \lg 3\right)$ as a single logarithm to base 10. [2]

(b) Hence solve the equation $2\lg x - \left(\lg(x+6) + \lg 3\right) = 0$. [4]

3 Variables x and y are such that when $\sqrt[3]{y}$ is plotted against x^2, a straight line passing through the points (9, 8) and (16, 1) is obtained. Find y as a function of x. [4]

4 The polynomial $p(x) = mx^3 - 17x^2 + nx + 6$ has a factor $x - 3$. It has a remainder of -12 when divided by $x + 1$. Find the remainder when $p(x)$ is divided by $x - 2$. [6]

5 **(a)** **(i)** Write down, in ascending powers of x, the first three terms in the expansion of $(1+4x)^n$. Simplify each term. [2]

(ii) In the expansion of $(1+4x)^n(1-4x)$ the coefficient of x^2 is 6032. Given that $n > 0$, find the value of n. [3]

(b) Find the term independent of x in the expansion of $\left(\frac{x}{2}-\frac{8}{x^4}\right)^{10}$. [2]

6 **(a)** **(i)** A 5-digit number is to be formed from the seven digits 0, 1, 2, 3, 4, 5, 6. Each digit can be used at most once in any number and the number does not start with 0. Find the number of ways in which this can be done. [2]

(ii) Find how many of these 5-digit numbers are even. [3]

(b) A team of 7 people is to be selected from a group of 9 women and 6 men. Find the number of different teams that can be selected which include at least one man. [2]

(c) **(i)** Show that ${}^{n}C_3 + {}^{n}C_2 = \frac{1}{6}(n^3 - n)$ for $n \geqslant 3$. [5]

(ii) Hence solve the equation ${}^{n}C_3 + {}^{n}C_2 = 4n$ where $n \geqslant 3$. [2]

7 Variables x and y are such that $y = \dfrac{(1+\sin 3x)^4}{\sqrt{x}}$. Use differentiation to find the approximate change in y when x increases from 1.9 to $1.9+h$, where h is small. [6]

8 In this question, **i** is a unit vector due east and **j** is a unit vector due north. Distances are measured in kilometres and time is measured in hours.

At 09 00, ship A leaves a point P with position vector $5\mathbf{i}+16\mathbf{j}$ relative to an origin O. It sails with a constant speed of $6\sqrt{3}$ on a bearing of 120°.

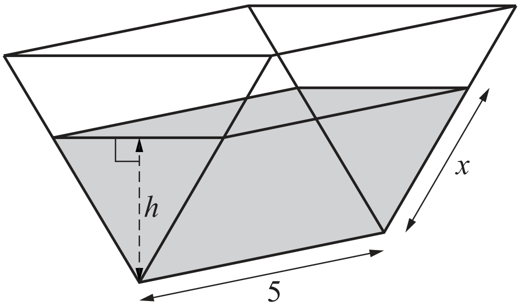
(a) Show that the velocity vector of A is $9\mathbf{i}-3\sqrt{3}\mathbf{j}$. [2]

(b) Find the position vector of A at 12 00. [1]

(c) At 11 00 ship B leaves a point Q with position vector $29\mathbf{i}+16\mathbf{j}$. It sails with constant velocity $-12\sqrt{3}\mathbf{j}$. Write down the position vector of B, t hours after it starts sailing. [1]

(d) Find the distance between the two ships at 12 00. [3]

9 In this question all lengths are in metres.



The diagram shows a water container in the shape of a triangular prism. The depth of water in the container is h. The container has length 5. The water in the container forms a prism with a uniform cross-section that is an equilateral triangle of side x.

(a) Show that the volume, V, of the water is given by $V = \dfrac{5\sqrt{3}h^2}{3}$. [4]

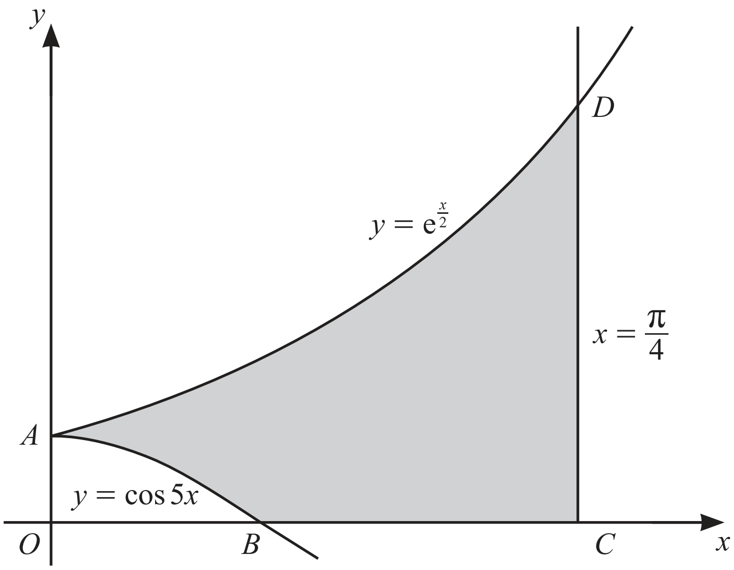
(b) Water is pumped into the container at a rate of 0.5 m^3 per minute. Find the rate at which the depth of the water is increasing when the depth of the water is 0.1 m. [4]

10 **(a)** Differentiate $x\ln x - 2x$ with respect to x. Simplify your answer. [2]

(b) A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(\frac{x+1}{\sqrt{x}}\right)^2$. It is given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^2}{2} + 2\mathrm{e}$ at the point $\left(\mathrm{e}, \frac{\mathrm{e}^3}{6} + \mathrm{e}^2\right)$.
Using your answer to **part (a)**, find the exact equation of the curve. [8]

Question 11 is printed on the next page.

11



The diagram shows part of the curves $y = e^{\frac{x}{2}}$ and $y = \cos 5x$ and part of the line $x = \frac{\pi}{4}$. The curves intersect at A. The curve $y = \cos 5x$ cuts the x-axis at B. The line $x = \frac{\pi}{4}$ cuts the x-axis at C and the curve $y = e^{\frac{x}{2}}$ at D. Find the exact area of the shaded region, $ABCD$. [7]